

Corrigé TE fractions rationnelles 1

Ex 1

$$a) \frac{x^3 - 16x}{x^4 - 8x^3 + 16x^2} = \frac{x(x^2 - 16)}{x^2(x^2 - 8x + 16)} = \frac{x(x+4)(x-4)}{x^2(x-4)^2} = \frac{x+4}{x(x-4)} \quad 5p$$

$$b) \frac{25 - 5x}{x^2 + x - 30} = \frac{5(5-x)}{(x+6)(x-5)} = \frac{-5(x-5)}{(x+6)(x-5)} = \frac{-5}{(x+6)} \quad 5p$$

Ex 2

$$\begin{array}{r} 2x^4 - 4x^3 + 2x^2 \\ -2x^4 \qquad \qquad + x^2 \\ \hline -4x^3 + x^2 \\ +4x^3 \qquad \qquad + 2x \\ \hline \qquad \qquad \qquad x^2 + 2x \\ -x^2 \qquad \qquad + \frac{1}{2} \\ \hline \qquad \qquad \qquad 2x - \frac{1}{2} \end{array}$$

$$2x^2 + 1$$

$$x^2 - 2x + \frac{1}{2}$$

$$Q(x) = x^2 - 2x + \frac{1}{2}$$

$$R(x) = 2x - \frac{1}{2}$$

4+2p

$$2x^4 - 4x^3 + 2x^2 = (2x^2 + 1)(x^2 - 2x + \frac{1}{2}) + 2x - \frac{1}{2}$$

1p

Ex 3

Horner avec $x = 11$:

$$\begin{array}{cccc} 2 & + 11 & - 380 & + 187 \\ \downarrow & \nearrow & \downarrow & \nearrow \\ 2 & 33 & -17 & \boxed{0} \end{array}$$

3p

$$\Rightarrow P(x) = (x-11)(2x^2 + 33x - 17)$$

$$= (x-11) \cdot 2(x - \frac{1}{2})(x+17)$$

$$= \underline{\underline{(x-11)(2x-1)(x+17)}}$$

$$\Delta = 33^2 - 4 \cdot 2 \cdot (-17) = 1225$$

$$\sqrt{\Delta} = 35$$

$$x = \frac{-33 \pm 35}{4} = \begin{cases} + \frac{1}{2} \\ -17 \end{cases}$$

5p

Ex 4

Candidats : $x=1$: $1 - 43 - 42 \neq 0$

$x=-1$: $-1 + 43 - 42 = 0 \checkmark$

1p

Horner avec $x = -1$

(-1)

1	0	-43	-42
↓	↗	↓	↗
1	-1	-42	0

2p

$\Rightarrow (x+1)(x^2 - x - 42) = 0$

1p

$(x+1)(x-7)(x+6) = 0$

\downarrow \downarrow \downarrow
 -1 7 -6

$S = \{-6; -1; 7\}$

2p

Ex 5

$P(-1) = (-1)^{100} + (-1)^{51} + (-1)^2 + (-1) = 1 - 1 + 1 - 1 = 0 \checkmark$

\Rightarrow oui, le reste est nul \Rightarrow divisible par $(x+1)$

3p