Corrigé processus exponentiels

Ex 1

a) $f(n) = 2^{n-1}$

b) $f(64) = 2^{63} \cong 9 \cdot 10^{18}$
(9 milliard de milliards)

$n = 1$	:	$1 = 2^0$
$n = 2$	:	$1 \cdot 2 = 2^1$
$n = 3$	:	$2 \cdot 2 = 2^2$

c) $2^{n-1} = 1'000'000 \quad | \log(\cdot)$

$\log(2^{n-1}) = \log(1'000'000)$

$(n-1) \cdot \log(2) = 6$

$n - 1 = \dfrac{6}{\log(2)}$

$n = \dfrac{6}{\log(2)} + 1 \cong 20,93 \quad \Rightarrow$ la 21^e case

Ex 2

a) $N(t) = 50 \cdot 1,6^t$

b) $N(8) = 50 \cdot 1,6^8 \cong 2147,48$
$\Rightarrow$ 2147 lapins

$n = 0$	:	50
$n = 1$	:	$50 + \dfrac{60}{100} \cdot 50 = 50 \cdot 1,6$
$n = 2$	:	$50 \cdot 1,6^2$

c) $50 \cdot 1,6^t = 20'000 \quad | : 50$

$1,6^t = 400 \quad | \log(\cdot)$

$\log(1,6^t) = \log(400)$

$t \cdot \log(1,6) = \log(400)$

$t = \dfrac{\log(400)}{\log(1,6)} \cong 12,75 \quad \Rightarrow$ au cours de la 12^e année

Ex 3

a) $t = 0$ en 1970

$S(t) = 6{,}7 \cdot 0{,}98^{t/5}$

$t = 0$:	$6{,}7$ mio
$t = 5$:	$6{,}7 - \frac{2}{100} \cdot 6{,}7 = 6{,}7 \cdot 0{,}98$
$t = 10$:	$6{,}7 \cdot 0{,}98^2$
$t = 15$:	$6{,}7 \cdot 0{,}98^3$

b) 1970 - 2013 : 43 ans

$S(43) = 6{,}7 \cdot 0{,}98^{43/5} \approx 5{,}6$ millions de km^2

c) $6{,}7 \cdot 0{,}98^{t/5} = 4{,}69 \quad | : 6{,}7$

$0{,}98^{t/5} = 0{,}7 \quad | \log(\cdot)$

$\log(0{,}98^{t/5}) = \log(0{,}7)$

$\frac{t}{5} \log(0{,}98) = \log(0{,}7) \quad | \cdot \frac{5}{\log(0{,}98)}$

$t = \frac{5 \cdot \log(0{,}7)}{\log(0{,}98)} \approx 88{,}27 \Rightarrow$ 89 ans après 1970

$\Rightarrow$ Au début 2059

Ex 4

a) $t = 0$ il y a 11 ans : 3260 $\xrightarrow{\cdot 1{,}65}$ $t = 11$ aujourd'hui : $3260 + 2119 = 5379$

$f(t) = 3260 \cdot 1{,}65^{t/11}$

$f(11+7) = f(18) = 3260 \cdot 1{,}65^{18/11} \approx 7397{,}74 \Rightarrow$ env. 7398

b) $3260 \cdot 1{,}65^{t/11} = 10'001 \quad | : 3260$

$1{,}65^{t/11} = \frac{10'001}{3260} \quad | \log(\cdot)$

$\frac{t}{11} \cdot \log(1{,}65) = \log\left(\frac{10'001}{3260}\right) \quad | \cdot \frac{11}{\log(1{,}65)}$

$t = \frac{11 \cdot \log\left(\frac{10'001}{3260}\right)}{\log(1{,}65)} \approx 24{,}62$

$\Rightarrow$ Dans 25 années complètes

Ex 5

a) Dans 20 ans, les 200 mio de jeunes formeront 100 mio de couples et engendreront 100 mio d'enfants

$t = 0$: 200 mio entre 20 et 40

$t = 20$: 100 mio

$t = 40$: 50 mio

$N(t) = 200 \cdot \left(\frac{1}{2}\right)^{t/20}$

b) $N(50) = 200 \left(\frac{1}{2}\right)^{50/20} \approx 35{,}4$ mio

c) $200 \cdot \left(\frac{1}{2}\right)^{t/20} = 10 \quad | : 200$

$\left(\frac{1}{2}\right)^{t/20} = \frac{1}{20} \quad | \log(\cdot)$

$\frac{t}{20} \cdot \log\left(\frac{1}{2}\right) = \log\left(\frac{1}{20}\right) \quad | \cdot \frac{20}{\log(1/2)}$

$t = \frac{20 \cdot \log(1/20)}{\log(1/2)} \approx 86{,}44 \Rightarrow$ Dans 87 ans

Ex 6

a) $t = 0$: 75 mg

$t = 1$: $75 - \frac{30}{100} 75 = 75 \cdot 0{,}7$

$t = 2$: $75 \cdot 0{,}7^2$

$Q(t) = 75 \cdot 0{,}7^t$

b) $Q(4) = 75 \cdot 0{,}7^4 \approx 18$ mg

c) $75 \cdot 0{,}7^t = 0{,}75 \quad | : 75$

$0{,}7^t = \frac{1}{100} \quad | \log(\cdot)$

$\log(0{,}7^t) = \log(0{,}01)$

$t \cdot \log(0{,}7) = \log(0{,}01) \quad | : \log(0{,}7)$

$t = \frac{\log(0{,}01)}{\log(0{,}7)} \approx 12{,}9$ heures

Ex 7

$t = 0$: 50 kg $\xrightarrow{\cdot 2/5}$ $t = 5$: 20 kg

$$Q(t) = 50 \cdot \left(\frac{2}{5}\right)^{t/5}$$

90% dissout $\Rightarrow$ reste 10% de la quantité initiale $\Rightarrow$ 5 kg

$$50 \cdot \left(\frac{2}{5}\right)^{t/5} = 5 \quad | : 50$$

$$\left(\frac{2}{5}\right)^{t/5} = 0{,}1 \quad | \log(\cdot)$$

$$\log\left(\left(\frac{2}{5}\right)^{t/5}\right) = \log(0{,}1)$$

$$\frac{t}{5} \cdot \log\left(\frac{2}{5}\right) = -1 \quad | \cdot \frac{5}{\log(2/5)}$$

$$t = \frac{-5}{\log(2/5)} \approx 12{,}56 \Rightarrow \text{encore } 7{,}56 \text{ h, soit } 7\text{h } 34$$

Ex 8

$$2 \cdot 2^{t/18} = 6 \cdot 2^{t/27} \quad | : 2$$

$$2^{t/18} = 3 \cdot 2^{t/27} \quad | : 2^{t/27}$$

$$\frac{2^{t/18}}{2^{t/27}} = 3$$

$$2^{t/18 - t/27} = 3$$

$$2^{t/54} = 3 \quad | \log(\cdot)$$

$$\log\left(2^{t/54}\right) = \log(3)$$

$$\frac{t}{54} \log(2) = \log(3) \quad | \cdot \frac{54}{\log(2)}$$

$$t = \frac{54 \log(3)}{\log(2)} \approx 85{,}59 \text{ ans}$$