2M - Révisions - Racines et puissances - Solutions

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Ex 1 :

a)
$$\begin{array}{r|l} 45 & 5 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 75 & 5 \\ 15 & 5 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

Alors $\sqrt{45} - \sqrt{75} + 2\sqrt{72}$
$$= 3\sqrt{5} - 5\sqrt{3} + 2\cdot 2\cdot 3\sqrt{2}$$
$$= \underline{\underline{3\sqrt{5} - 5\sqrt{3} + 12\sqrt{2}}}$$

b)
$$\frac{(\sqrt{2} - \sqrt{3})}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6} - 3}{3}$$

c)
$$\begin{array}{r|l} 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 80 & 2 \\ 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

Alors $(2\sqrt{48} - \sqrt{5})(\sqrt{80} + \sqrt{3}) = (2\cdot 4\sqrt{3} - \sqrt{5})(4\sqrt{5} + \sqrt{3})$
$$= (8\sqrt{3} - \sqrt{5})(4\sqrt{5} + \sqrt{3}) = 32\sqrt{15} + 8\cdot 3 - 4\cdot 5 - \sqrt{15}$$
$$= \underline{\underline{31\sqrt{15} + 4}}$$

d)
$$\frac{(1-\sqrt{3})}{(1+\sqrt{3})} \cdot \frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{1 - 2\sqrt{3} + 3}{1-3} = \frac{4 - 2\sqrt{3}}{-2} = -2 + \sqrt{3} = \underline{\underline{\sqrt{3} - 2}}$$

e)
$$\begin{array}{r|l} 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

et $8 = 2^3$ donc $3\sqrt{8} - \sqrt{32}(\sqrt{2} - 1)$
$$= 3\cdot 2\sqrt{2} - 4\sqrt{2}(\sqrt{2} - 1)$$
$$= 6\sqrt{2} - 4\cdot 2 + 4\sqrt{2} = \underline{\underline{10\sqrt{2} - 8}}$$

f) $\sqrt[4]{2} \cdot \sqrt{2} \cdot \sqrt[8]{4^5}$
$$= \sqrt[4]{2} \cdot \sqrt{2} \cdot \sqrt[8]{2^{10}} = \sqrt[4]{2} \cdot \sqrt[4]{2^2} \cdot \sqrt[4]{2^5} = \sqrt[4]{2^8} = 2^2 = \underline{4}$$

g)
$$\frac{4\sqrt{18}}{\sqrt{27}} = \frac{4\sqrt{9\cdot 2}}{\sqrt{9\cdot 3}} = \frac{4\cdot 3\sqrt{2}}{3\sqrt{3}} = \frac{4\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \underline{\underline{\frac{4\sqrt{6}}{3}}}$$

h) $\sqrt{7} \cdot \sqrt{\frac{1}{21}} \cdot \sqrt{\frac{3}{16}}$
$$= \sqrt{\frac{7\cdot 3}{21\cdot 16}} = \sqrt{\frac{1}{16}}$$
$$= \underline{\underline{\frac{1}{4}}}$$

i)
$$\frac{2+\sqrt{3}}{2-\sqrt{2}} \cdot \frac{(2+\sqrt{2})}{(2+\sqrt{2})} = \frac{4 + 2\sqrt{2} + 2\sqrt{3} + \sqrt{6}}{4-2} = \underline{\underline{\frac{4 + 2\sqrt{2} + 2\sqrt{3} + \sqrt{6}}{2}}}$$

j) $\sqrt{2} + \sqrt[3]{3^2}\,\sqrt[4]{3}\,\sqrt[12]{3}$
$$= \sqrt{2} + \sqrt[12]{3^8}\,\sqrt[12]{3^3}\,\sqrt[12]{3} = \sqrt{2} + \sqrt[12]{3^{12}} = \sqrt{2} + 3 = \underline{\underline{3 + \sqrt{2}}}$$

k)
$$\frac{5-\sqrt{5}}{5+\sqrt{5}} \cdot \frac{5-\sqrt{5}}{5-\sqrt{5}} = \frac{25 - 10\sqrt{5} + 5}{25-5} = \frac{30 - 10\sqrt{5}}{20} = \underline{\underline{\frac{3-\sqrt{5}}{2}}}$$

l)
$$\sqrt[3]{9} \cdot \sqrt{125} \cdot \sqrt[6]{15^3} = \sqrt[3]{3^2} \cdot \sqrt{5^3} \cdot \sqrt[6]{15^3} = \sqrt[6]{3^4}\,\sqrt[6]{5^9}\,\sqrt[6]{5^3\cdot 3^3} = \sqrt[6]{3^7\, 5^{12}}$$
$$= 5^2 \cdot 3\sqrt[6]{3} = \underline{\underline{75\sqrt[6]{3}}}$$

m)
$$\sqrt{18} - 5\sqrt{48} + \sqrt{54}(2 - \sqrt{6}) = 3\sqrt{2} - 20\sqrt{3} + 3\sqrt{6}(2 - \sqrt{6}) = \underline{\underline{3\sqrt{2} - 20\sqrt{3} + 6\sqrt{6} - 18}}$$

Ex 2 :

a) $(2^2)^{-\frac{1}{2}}\cdot(3^3)^{\frac{2}{3}}+\left(\frac{125}{10^3}\right)^{1/3}\cdot\left(\frac{32}{10^5}\right)^{1/5} = 2^{-1}\cdot 3^2+\left(\frac{5^3}{10^3}\right)^{1/3}\cdot\left(\frac{2^5}{10^5}\right)^{1/5}$

$= \frac{9}{2}+\frac{5}{10}\cdot\frac{2}{10}=\frac{9}{2}+\frac{1}{10}=\frac{45}{10}+\frac{1}{10}=\frac{46}{10}=\underline{\underline{\frac{23}{5}}}$

b) $\sqrt[6]{4^3}=\sqrt[6]{2^6}=\underline{\underline{2}}$ c) $\sqrt[4]{\frac{64}{10^4}}=\sqrt[4]{\frac{2^6}{10^4}}=\frac{2}{10}\sqrt[4]{2^2}=\frac{2\sqrt{2}}{10}=\underline{\underline{\frac{\sqrt{2}}{5}}}$

d) $\frac{1}{3^3}-\frac{1}{2^2}\cdot\frac{1}{4}=\frac{1}{27}-\frac{1}{16}=\frac{16-27}{432}=\underline{\underline{-\frac{11}{432}}}$

e) $\sqrt[4]{128}=\sqrt[4]{2^7}=\sqrt[4]{2^4\cdot 2^3}=\underline{\underline{2\sqrt[4]{8}}}$

f) $32^{1/2}=2^{5/2}=2^2\sqrt{2}=\underline{\underline{4\sqrt{2}}}$ g) $\sqrt[3]{\sqrt{512}}=\sqrt[6]{2^9}=\sqrt{2^3}=\underline{\underline{2\sqrt{2}}}$

h) $(-2)^{-4}\cdot 3^4=\frac{1}{(-2)^4}\cdot 3^4=\underline{\underline{\frac{81}{16}}}$ i) $\left(\frac{3}{7}\right)^{-2}=\frac{7^2}{3^2}=\underline{\underline{\frac{49}{9}}}$

j) $3^1=\underline{\underline{3}}$ k) $(5^3)^{-\frac{1}{2}}=\frac{1}{5^{3/2}}=\frac{1}{5\sqrt{5}}=\underline{\underline{\frac{\sqrt{5}}{25}}}$

l) $a^{7/3}=\sqrt[3]{a^7}=\sqrt[3]{a^6\cdot a}=\underline{\underline{a^2\cdot\sqrt[3]{a}}}$

m) $3\cdot(3^3)^{-\frac{1}{3}}+16^{1/2}\cdot\left(\frac{16}{10^4}\right)^{1/4}=3\cdot 3^{-1}+4\cdot\frac{2}{10}=1+\frac{4}{5}=\underline{\underline{\frac{9}{5}}}$

n) $\left(2^{-\frac{2}{3}}\cdot 2^{2/3}\cdot 2^{6/3}+(2^4)^{3/4}\right)^{1/2}=\left(2^2+2^3\right)^{1/2}=\sqrt{4+8}=\sqrt{12}=\underline{\underline{2\sqrt{3}}}$

Ex 3

$A=\frac{a^{-2}b^{-3}\,a^{-15}b^{-5}}{a^{-12}\,b^{-10}}=a^{(-2-15+12)}\,b^{(-3-5+10)}=a^{-5}b^2=\underline{\underline{\frac{b^2}{a^5}}}$

avec $a=3^4$ et $b=3^{11}$

$A=\frac{3^{22}}{3^{20}}=3^2=\underline{\underline{9}}$